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HEAT EXCHANGER DESIGN WITH VARIABLE TRANSFER COEFFICIENTS FOR CROSSFLOW AND MIXED FLOW ARRANGEMENTS*

WILFRIED ROETZEL†
Chemical Engineering Research Group, Council for Scientific and Industrial Research, Pretoria, South Africa

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Abstract—In special cases of crossflow the mean overall heat-transfer coefficient can be calculated, when either only laminar length effects or only temperature dependences are present. With one laminar stream the mean value can be calculated as for cocurrent and countercurrent flow and the result is valid for any flow arrangement. For the case of two laminar streams a solution is presented which is valid for any crossflow arrangement.

For only temperature-dependent heat transfer and crossflow with only one row a special method is presented. According to this method heat transfer must be calculated at two points if it depends only on the fluid temperature inside the tube, and at four points if it is a function of both fluid temperatures.

Further, a more general approximation method is described for the calculation of the mean overall heat-transfer coefficient and the overall pressure drop, which is valid for any flow arrangement and combined length and temperature effects. This approach was developed from a previous pure counterflow method; the heat transfer and pressure drop must be calculated at two points.

The general method was tested with examples for which reliable solutions (special crossflow cases and cocurrent flow) are available and very good agreement was obtained.

NOMENCLATURE

- A, dimensionless group, defined by (16);
- B, integration constant;
- a, dimensionless group, defined by (7);
- $\dot{C}$, mass flow rate times specific heat (of the hot fluid);
- F, total heat-transfer area;
- f, variable, defined by (25);
- g, variable, defined by (26);
- I, variable representing an integral;
- K, local overall heat-transfer coefficient;
- L, flow length (of hot stream) in one pass;
- l, variable flow length calculated from the tube inlet (flat profiles);
- n, exponent;
- p, pressure (of hot stream);
- T, temperature (of hot stream);
- t, corrected reference temperature, defined by (36);
- V, overall flow length correction factor, defined by (6);
- w, wall resistance;
- x, dummy variable.

Greek symbols

- α, local heat-transfer coefficient (of hot stream);
- β, γ, exponents;
- Δ, finite difference;
- δ, local flow length correction (of hot stream);
- Θ, dimensionless temperature (of hot stream);
- ξ, dimensionless flow path of hot stream, defined by (1);
- ϕ, dimensionless flow path of cold stream, defined by (1);
- ψ, reference temperature correction (of hot stream).

Subscripts

- 0, $\xi = 0$, inlet of hot stream;
- 1, $\xi = 1$, outlet of hot stream;
- ϕ, inlet of cold stream, $= 0$;
- ϕ, outlet of cold stream, $= 1$;
- I, II, reference points for Gaussian integration over ξ-co-ordinate;
- i, counter for I and II;
- a, b, reference points for Gaussian integration over ϕ-co-ordinate;
- c, counterflow;
- in, out, at inlet or outlet, respectively;
- (in), (out), determined only with the pressure of inlet or outlet, respectively;
- L, laminar;
- M, correct mean value for heat transfer;

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†Present address: Bayer AG, Verfahrenstechnik R150, 415 Krefeld 11, Germany.

$x, y, z,$ identifiers for distinction of different integrals.

Superscripts

$'$, cold stream, for one row crossflow unmixed or outside the tubes;

$\bar{}$, mean value through adiabatic mixing;

$\tilde{}$, area average mean value at constant temperature (only length effect included).

1. INTRODUCTION

FOR THE cost-optimized design of heat exchangers with the computer [1] as well as for manual design calculations reliable but fast calculation methods for the mean overall heat-transfer coefficient and the overall pressure drop are required because, on the one hand, the conventional simple method using mean values of temperatures as reference temperatures can lead to undesirable errors in design and, on the other hand, numerical stepwise integrations are prohibitively time consuming.

Various methods for estimating the mean heat-transfer coefficient [2–5] and the overall pressure drop [6] have been presented taking into account the variation of the heat-transfer coefficients and the differential pressure drop with temperature and/or length of flow path.

However, considering the temperature effect, all methods were derived for pure cocurrent or countercurrent flow. In air-cooled heat exchangers, as applied frequently nowadays, the fin tube bundles are usually in crossflow and the counterflow methods are then only approximations, with the accuracy increasing with the number of passes. Similar considerations are valid also for shell and tube heat exchangers where true countercurrent flow is only a limiting case.

The current paper covers the gap between pure countercurrent flow and cocurrent flow and first some special cases of crossflow are considered.

2. CROSSFLOW WITH HEAT-TRANSFER COEFFICIENTS VARYING WITH EITHER TEMPERATURE OR LENGTH OF FLOW PATH

We consider a crossflow heat exchanger as shown in Fig. 1. The dimensionless flow paths of the hot stream $\dot{C}$ flowing inside the tubes in air coolers and of the cold stream $\dot{C}'$ are, respectively,

$$\xi = \frac{l}{L}, \qquad \phi = \frac{l'}{L'}. \tag{1}$$

The local overall heat-transfer coefficient

$$\frac{1}{K} = \frac{1}{\alpha} + w + \frac{1}{\alpha'}, \tag{2}$$

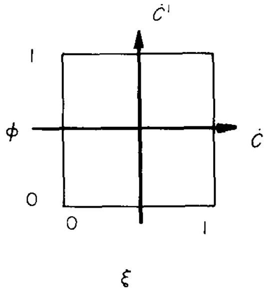

FIG. 1. Dimensionless flow paths ϕ and ξ and dimensionless heat-transfer area "1" for crossflow.

where each resistance is related to the same area. The heat-transfer coefficients α and α' are changing with both temperature and length of flow path but in many cases only one of both effects is decisive. For turbulent flow (and radiation) the temperature dependence is important but the flow length effect can be neglected [4]. On the other hand, with laminar flow the temperature effect is weak whereas the flow length effect is most important and the local heat-transfer coefficient can then be expressed by

$$\begin{aligned} \alpha_L &= \tilde{\alpha}_L \cdot \frac{2}{3} \cdot \xi^{-1/3} = \tilde{\alpha}_L \cdot \frac{1}{\delta} \\ \alpha'_L &= \tilde{\alpha}'_L \cdot \frac{2}{3} \cdot \phi^{-1/3} = \tilde{\alpha}'_L \cdot \frac{1}{\delta'} \end{aligned} \tag{3}$$

where in the actual case $\tilde{\alpha}_L$ and $\tilde{\alpha}'_L$ are functions of the fluid and wall surface temperatures.

According to the conventional method one calculates the mean overall heat-transfer coefficient with the area average heat-transfer coefficients (arrived at by integration at constant wall and fluid temperature) which are obtained from the usual heat-transfer equations.

$$\frac{1}{\tilde{K}} = \frac{1}{\tilde{\alpha}} + w + \frac{1}{\tilde{\alpha}'} \tag{4}$$

with subscript L for the alphas in laminar flow. The true mean overall heat-transfer coefficient K_M, however, needed for the design of a crossflow heat exchanger is (for constant $\dot{C}$ and $\dot{C}'$)

$$K_M = \int_{\phi=0}^{\phi=1} \int_{\xi=0}^{\xi=1} K \, d\phi \, d\xi. \tag{5}$$

The mean value K_M shall be determined for some special cases.

2.1 *Only laminar length effects*

We consider the case that $\tilde{\alpha}_L$, $\tilde{\alpha}'_L$ and all other resistances are constant and thus in (5) K is only a direct function of ϕ and/or ξ according to (2) and (3).

As in [4] and [5] we introduce the ratio

$$V = \frac{K_M}{\tilde{K}} \tag{6}$$

2.1.1 *One stream laminar.* When the hot stream $\dot{C}$ is in laminar flow α in (2) must be replaced by α_L according to (3). In (5), substituting K according to (2) and introducing (5) and (4) into (6) gives, with the dimensionless group

$$a = \frac{\tilde{\alpha}_L}{\tilde{K}} - 1 \tag{7}$$

the following integral (the integration over ϕ, (5), gives the factor one):

$$V = \int_{\xi=0}^{\xi=1} \frac{1+a}{\frac{3}{2}\xi^{1/3}+a} \, . \, d\xi. \tag{8}$$

Integration gives [6]:

$$V = (1+a)\left[1 - \frac{4}{3}a + \frac{8}{9}.a^2.\ln\left(1+\frac{3}{2a}\right)\right]. \tag{9}$$

For $a \to \infty$ and $a \to 0$ the correction $V \to 1$ as expected from (8).

For the other case that $\dot{C}'$ is laminar a in (9) must be replaced by a' according to (7) with $\tilde{\alpha}'_L$ instead of $\tilde{\alpha}_L$.

The correction factor V according to (9) is equal to that of cocurrent, countercurrent and mixed parallel-countercurrent flow derived by Peters [5]. In contradiction to Peters' opinion [5], the correction factor is also exactly true for crossflow, as shown by our derivation.

The integration according to (5) could be done over separate partial areas $\Delta\phi$, where in each $\Delta\phi$–ξ strip the flow direction can be selected arbitrarily and the same result would be obtained as given by (9). Thus (9) is valid exactly for crossflow with any number of passes and any but equal number of rows (of equal tube diameter and length) in each pass where the local $\tilde{\alpha}_L$ or $\tilde{\alpha}'_L$ is constant over the heat-transfer area. Equation (9) is exact also for any flow arrangement where $\tilde{\alpha}_L$ or $\tilde{\alpha}'_L$ in each tube section through which the laminar stream flows parallel or in series has the same value.

For cocurrent flow (9) is also valid when both streams are laminar; then one has to consider the sum of both local laminar heat-transfer resistances as one laminar heat-transfer resistance. For crossflow, however, two laminar streams have to be treated in the following way.

2.1.2 *Both streams laminar and zero wall resistance.* In the same way as leads to (8) but introducing both laminar heat-transfer coefficients with zero wall resistance:

$$V = \int_{\phi=0}^{\phi=1} \int_{\xi=0}^{\xi=1} \frac{\frac{2}{3}(1+a)}{\xi^{1/3}+a.\phi^{1/3}} . d\phi . d\xi \tag{10}$$

where now $a = 1/a'$ because $w = 0$. Exchanging ϕ and ξ and by replacing a with a' shows that V is equal for a or $a' = 1/a$, respectively.

Integrating over ξ from 0 to 1 with constant ϕ gives according to (9)

$$V = (1+a).\int_{\phi=0}^{\phi=1}\left[1 - 2a.\phi^{1/3} + 2a^2.\phi^{2/3}.\ln\left(1+\frac{1}{a.\phi^{1/3}}\right)\right].d\phi. \tag{11}$$

The integration over ϕ from 0 to 1 gives (see Appendix):

$$V = \frac{6}{5} - \frac{3}{5}.\left(a+\frac{1}{a}\right) + \frac{6}{5}(a^2+a^3).\ln\left(1+\frac{1}{a}\right) + \frac{6}{5}.\left(\frac{1}{a^2}+\frac{1}{a^3}\right).\ln(1+a) - \frac{6}{5}\left(a^2+\frac{1}{a^2}\right). \tag{12}$$

Equation (12) is symmetric for a and $a' = 1/a$. For $a \to 0$ or $a \to \infty$ (12) yields $V = 1$. As in the case of one laminar stream the twofold integration can be done over separate sections $\Delta\xi$ or (and) $\Delta\phi$, where again the flow direction can be inverted in individual strips leading to (12). This equation is valid for any kind of crossflow provided the local values of $\tilde{\alpha}_L$ and $\tilde{\alpha}'_L$ are constant over the heat-transfer area and the flow directions of both fluids are perpendicular to each other.

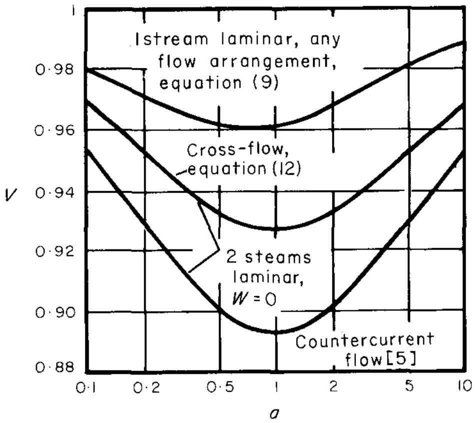

FIG. 2. Correction factor $V = K_M/\tilde{K}$ for laminar flow length effect as function of $a = (\tilde{\alpha}_L/\tilde{K}) - 1$.

Figure 2 shows (9) and (12) and also the corresponding correction factor for pure counterflow and zero wall resistance from Peters [5]. The correction factor for two laminar streams and zero wall resistance for crossflow lies between the corresponding factor for counterflow and that of cocurrent flow ($V = 1$ for $w = 0$).

2.2 Temperature effects only; one-row crossflow arrangement

Here we must distinguish between different crossflow arrangements and we consider the rather uncomplicated and, with respect to mean temperature difference, most disadvantageous case of one row one pass.

In air-cooled fin tube crossflow heat exchangers a hot liquid is usually inside the tubes. The fluid in the tubes is considered to be completely mixed in any cross section. T is the temperature of the mixed fluid. T' is the temperature of the unmixed fluid (usually cold air) outside the tubes. We now introduce the dimensionless temperature of the mixed fluid

$$\Theta = \frac{T - T'_{in}}{T_{in} - T'_{in}}. \tag{13}$$

Replacing the variable temperatures T and Θ in (13) by T' and Θ' gives the dimensionless temperature of the unmixed fluid.

The dimensionless mean temperature difference is:

$$\Delta\Theta_M = \frac{\Delta T_M}{T_{in} - T'_{in}}. \tag{14}$$

The change of temperatures along the flow paths is shown qualitatively in Fig. 3.

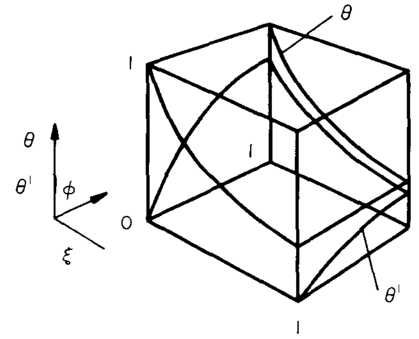

FIG. 3. Distribution of the dimensionless temperatures Θ and Θ' over the dimensionless heat-transfer area for one row crossflow.

For the mixed fluid $\dot{C}$ at the inlet $\xi = 0$ and $\Theta = \Theta_0 = 1$. At the outlet $\xi = 1$ and $\Theta = \Theta_1$ with $T = T_{out}$. For the unmixed fluid $\dot{C}'$ at the inlet $\phi = 0$ and $\Theta' = \Theta'_{\phi=0} = 0$. At the outlet locally (changing with ξ) $\phi = 1$ and $\Theta' = \Theta'_{\phi=1}$ with $T' = T'_{out}$. Considering the adiabatic mixing outlet temperature which is usually given $\phi = 1$ and $\Theta' = \bar{\Theta}'_{\phi=1}$ with $T' = \bar{T}'_{out}$.

2.2.1 *Local heat transfer depending only on the temperature of the mixed fluid.* With air-cooled heat exchangers the temperature dependence of the liquid inside the tubes is more decisive than that of the air outside the fin tubes. Therefore (and also for simplification), we first consider the case that the local overall heat-transfer coefficient is only a function of the temperature T or Θ, respectively. Later in this paper the method is extended to allow also for variation with T'.

We make the reasonable simplifying assumption that $\dot{C}$ and $\dot{C}'$ are independent of temperature.

2.2.1.1 *Equation for the true mean overall heat-transfer coefficient*—The heat which is transferred through the small area $F\,.\,d\xi$

$$-\dot{C}\,.\,d\Theta = \dot{C}'\,.\,\Theta'_{\phi=1}\,.\,d\xi = K\,.\,F\,.\,d\xi\,.\,\frac{\Theta'_{\phi=1}}{\ln\dfrac{\Theta}{\Theta - \Theta'_{\phi=1}}} \tag{15}$$

where K is a function only of Θ. From the middle and righthand part of (15) $\Theta'_{\phi=1}$ can be expressed as a function of Θ and A, the latter defined by

$$A = \frac{K\,.\,F}{\dot{C}'}. \tag{16}$$

Substituting accordingly in (15) gives

$$-\dot{C}\,.\,d\Theta = \dot{C}'\,.\,\Theta\,.\,(1 - e^{-A})\,.\,d\xi. \tag{17}$$

Solving for $\dot{C}'/\dot{C}\,.\,d\xi$, introducing $\ln\Theta$ as a variable [4], [6] and integrating from $\xi = 0$ to 1 and Θ from 1 to Θ_1 gives:

$$\frac{\dot{C}'}{\dot{C}} = -\int_{\Theta=1}^{\Theta=\Theta_1} \frac{d\ln\Theta}{1 - e^{-A}} = -\ln\Theta_1\,.\,\frac{1}{1 - e^{-A_M}}. \tag{18}$$

This equation *defines* A_M as the true mean value over the total area F, determined according to (16) with the true mean value of the overall heat-transfer coefficient K_M [according to (5)].

In a *design* procedure the mean value A_M is given because Θ_1, $\dot{C}'$ and $\dot{C}$ are given. From the first and last parts of (18) we find by solving for A_M

$$A_M = -\ln\left[1 + \frac{\dot{C}}{\dot{C}'}\,.\,\ln\Theta_1\right]. \tag{19}$$

On the other hand, a total heat balance gives

$$A_M = \frac{\bar{\Theta}'_{\phi=1}}{\Delta\Theta_M}. \tag{20}$$

Substituting A_M in (19) according to (20) gives the known [7], [8] equation for the mean temperature difference

$$\Delta\Theta_M = \frac{-\bar{\Theta}'_{\phi=1}}{\ln\left[1 + \dfrac{\dot{C}}{\dot{C}'}\,.\,\ln\Theta_1\right]} \tag{21}$$

which is needed for the calculation of the heat-transfer area. The corresponding true mean value K_M we have to determine with the middle part of (18) by substituting A according to

$$A = A_M\,.\,\frac{K}{K_M} \tag{22}$$

and integrating numerically with an estimated value of K_M, the true value of A_M and the temperature dependent changing value of K. If one has estimated the true value of K_M the integration fulfils (18). This trial and error method is very complicated and time consuming and a faster method is derived below.

2.2.1.2 *Integration using a two-point Gauss method*—We can obtain the first simplification by using the two-point integration method of Gauss, which method integrates exactly a polynomial function of third degree. According to [4] and [6], (18) becomes

$$\frac{1}{1-e^{-A_M}} = \frac{1}{2}\left(\frac{1}{1-e^{-A_I}} + \frac{1}{1-e^{-A_{II}}}\right) \tag{23}$$

where A_I and A_{II} must be evaluated according to (22) with K_I and K_{II}. These reference values of the local overall heat-transfer coefficient must be determined with the reference temperatures T_I and T_{II} according to

$$\begin{aligned}\Theta_I &= \Theta_I^{0\cdot 21132}\\ \Theta_{II} &= \Theta_I^{0\cdot 78868}\end{aligned} \tag{24}$$

and (13). Now one can find the wanted value K_M in A_I and A_{II} by estimating K_M until (23) is fulfilled.

However, the following iterative method is far better. It was found that (23) is also fulfilled with good approximation when A_M, A_I and A_{II} are multiplied with a factor close to one. This fact is applied in the following method where A_M, A_I and A_{II} are multiplied by the ratio K_M/K_M^*. K_M^* is an estimated value of K_M. Multiplying A_I and A_{II} with the ratio K_M/K_M^* gives A_I^* and A_{II}^*. The values K_M cancel by this multiplication and A_I^* and A_{II}^* must be determined according to (22) with the estimated value K_M^* instead of K_M and the true values of A_M and K_I or K_{II}. Now the r.h.s. of (23) can be calculated with A_I^* and A_{II}^* yielding the mean value on the l.h.s. $A_M^* = A_M . K_M/K_M^*$. By the multiplication with the ratio K_M/K_M^* K_M disappeared on the r.h.s. and appeared on the l.h.s. in A_M^*. Solving for K_M gives an improved value of K_M^* and the procedure can be repeated. The convergence is very good.

2.2.1.3 *Non-iterative approximation for the two-point mean value*—In order to find a good approximate value K_M^* to start with or eventually to bypass the iteration, the following approach was derived.

If one could find a function of A represented by the variable g which is a linear function of f

$$f = \frac{1}{1-e^{-A}} \tag{25}$$

then in analogy to (23) one could form the mean value g_M of A_M from the reference values g_I and g_{II} of A_I and A_{II}, resulting in the same A_M as by (23).

We now assume that the function

$$g = \left(\frac{1}{A}\right)^\beta \tag{26}$$

is approximately a linear function of f. If we now form the mean value of g_I and g_{II}, the unknown area F and the heat capacity $\dot{C}'$ in A [see (16)] cancel and

$$\left(\frac{1}{K_M}\right)^\beta = \frac{1}{2}\left[\left(\frac{1}{K_I}\right)^\beta + \left(\frac{1}{K_{II}}\right)^\beta\right]. \tag{27}$$

We now have to find a suitable value of β.

Close to A_M the linearity between f and g is fulfilled, if

$$\left(\frac{df}{dg}\right)_{A_M} = \left(\frac{df}{dg}\right)_{A_M+\Delta A} \tag{28}$$

or (after introducing the variables A) if

$$\frac{(df/dA)_{A_M}}{(df/dA)_{A_M+\Delta A}} = \frac{(dg/dA)_{A_M}}{(dg/dA)_{A_M+\Delta A}}. \tag{29}$$

Replacing f and g according to (25) and (26) gives:

$$\frac{e^{A_M+\Delta A}.(1-e^{-A_M-\Delta A})^2}{e^{A_M}.(1-e^{-A_M})^2} = \left(\frac{A_M+\Delta A}{A_M}\right)^{\beta+1}. \tag{30}$$

For the limiting case $\Delta A \to 0$ the following equation can be derived from (30):

$$\beta = A_M . \frac{e^{A_M}+1}{e^{A_M}-1} - 1. \tag{31}$$

For $A_M \to \infty$ $\beta = A_M - 1$ and for $A_M \to 0 (\dot{C}' \to \infty) \beta = 1$. In the latter limiting case T' is constant and this cross-flow method turns into the known cocurrent or countercurrent method [4].

The accuracy of using (27) and (31) is illustrated by Table 1. The calculated examples demonstrate clearly that the derived approach yields good approximate

Table 1. Validity of the approach using equations (27) and (31) (or $\beta = 1$)

A_I	A_{II}	A_M, equation (23)	β, equation (31)	$A_M(A = K)$, equation (27)	Error (rel.) (%)	Error for $\beta = 1$ (%)
0·2	0·4	0·26647	1·0118	0·26649	+0·007	+0·074
1·0	2·0	1·31055	1·2784	1·31283	+0·174	+1·738
5·0	10·0	5·68312	4·7219	5·74514	+1·091	+17·306
25·0	50·0	25·69315	24·6932	25·71170	+0·072	+29·736
0·1	1·0	0·18083	1·0054	0·18143	+0·335	+0·547
1·0	10·0	1·48982	1·3569	1·61465	+8·378	+22·040

values of K_M. In almost all practical applications these approximate values will be sufficiently accurate. Particularly when a computer is used one or two iterations should be done with this approximate value of K_M, just to make sure.

2.2.1.4 *Summary of procedure*—The true mean overall heat-transfer coefficient can be calculated, if the local overall transfer coefficient is a function only of the temperature T of the mixed stream (inside the tubes of one row) and if the heat capacities of both streams are constant.

From the known inlet and outlet temperatures first the dimensionless outlet temperature of the mixed stream Θ_I is calculated according to (13) with $T = T_{out}$. Then the dimensionless reference temperatures Θ_I and Θ_{II} according to (24) and the real temperatures T_I and T_{II} according to (13) with $T = T_I$ or T_{II}, respectively, are determined. Now the two local overall heat-transfer coefficients K_I and K_{II} are calculated using the reference temperatures T_I and T_{II} and (2).

The mean value A_M is calculated according to (19), the exponent β according to (31) and the mean value K_M according to (27). K_M thus obtained is a good approximate value of the wanted true mean overall heat-transfer coefficient. Taking this value as K_M^* it can still be improved by iterations described in section 2.2.1.2 using (22) and (23).

2.2.2 *Local heat-transfer coefficient depending on both fluid temperatures.* The method described above can be extended easily to the case that the local overall heat-transfer coefficient is a function also of the temperature T' of the unmixed stream (outside the fin tubes).

Instead of the constant values K_I and K_{II} which were independent of T' and ϕ, we now have to introduce the true mean values K_i with $i = \mathrm{I}$ and II, where T has the constant value T_i and T_i' is changing from $T'_{i,in}$ to $T'_{i,out}$. This case can be treated according to the two-point method described in [4]. For $i = \mathrm{I}$ and II:

$$\frac{1}{K_i} = \frac{1}{2}\left(\frac{1}{K_{i,a}} + \frac{1}{K_{i,b}}\right). \tag{32}$$

The local overall transfer coefficients must be determined with the temperatures T_i and $T'_{i,a}$ or $T'_{i,b}$ according to

$$\begin{aligned} \Theta'_{i,a} &= \Theta_i(1 - e^{-A_M \cdot K_i/K_M \cdot 0{\cdot}21132}) \\ \Theta'_{i,b} &= \Theta_i(1 - e^{-A_M \cdot K_i/K_M \cdot 0{\cdot}78868}) \end{aligned} \tag{33}$$

and (13).

Equation (33) shows that for the determination of the reference temperatures for T' which are needed for the calculation of the mean value K_i according to (32), this local mean value K_i and the wanted true mean value K_M must already be known. Thus iterations cannot be avoided. One must estimate or take from a previous calculation approximate values of K_i and K_M in order to determine the reference temperatures for T' according to (33). For the first step the ratio K_i/K_M could be taken as one.

Thus the true mean overall heat-transfer coefficient can also be calculated if the local overall heat-transfer coefficient depends on both fluid temperatures.

The accuracy of this method is as high as that of the two-point integration for which the high accuracy was proved in similar heat-transfer cases [4, 5]; it need not be tested here again.

3. GENERAL APPROXIMATION METHOD FOR ANY FLOW ARRANGEMENT AND COMBINED TEMPERATURE AND LENGTH EFFECTS

In practical cases for which a cost-optimized design has to be carried out, the special cases treated in the previous chapters will not arise very frequently and usually not in the pure form of either only length or only temperature effects. For cost-optimized design, where the flow arrangement changes during the optimization procedure and where combined length and temperature effects may occur, a more general, less accurate but simple method is very useful. A simple method can take the actual flow arrangements into account only approximately.

Previously, for the calculation of the mean overall heat-transfer coefficient and the pressure drop the flow arrangement was regarded as countercurrent flow [4, 6]. This assumption is reasonable because in most cases one tries to be close to pure countercurrent flow with its advantageous mean temperature difference. However, cases may arise in which this method is not accurate enough. Therefore, we extend the countercurrent flow method by slight variations to allow approximately also for any flow arrangement.

3.1 *Determination of reference temperatures*

3.1.1 *Pure counterflow.* For pure counterflow the reference temperatures of the hot stream $\dot{C}$ and the cold stream $\dot{C}'$ are determined [4, 6] by:

$$\begin{aligned} \Delta T_I &= (T_{in} - T'_{out})^{0{\cdot}78868} \,.\, (T_{out} - T'_{in})^{0{\cdot}21132} \\ \Delta T_{II} &= (T_{in} - T'_{out})^{0{\cdot}21132} \,.\, (T_{out} - T'_{in})^{0{\cdot}78868}. \end{aligned} \tag{34}$$

From the temperature differences and assuming constant specific heats we find the reference temperatures T_i and T_i' with $i = \mathrm{I}$ and II.

$$\frac{T_i - T_{out}}{T_{in} - T_{out}} = \frac{T_i' - T'_{in}}{T'_{out} - T'_{in}} = \frac{\Delta T_i - (T_{out} - T'_{in})}{(T_{in} - T'_{out}) - (T_{out} - T'_{in})}. \tag{35}$$

For $\dot{C} = \dot{C}'$ the righthand term of (35) turns to 0·78868 or 0·21132, respectively.

3.1.2 *Correction of the reference temperatures for other flow arrangements.* With fixed inlet and outlet temperatures pure counterflow yields the highest mean temperature difference. For any other flow arrangement the difference between the mean temperatures of the hot and the cold stream, averaged over the heat-transfer area, is smaller. Thus for any other flow arrangement the mean temperature of the hot fluid is usually lower than in the case of counterflow, and the mean temperature of the cold fluid is usually higher. This knowledge we apply when introducing a correction to the temperatures relevant in pure counterflow:

$$t_i = T_i - \psi_i \qquad t_i' = T_i' + \psi_i' \tag{36}$$

where the correction terms ψ_i and ψ_i' are usually both positive values; these terms should disappear in the limiting case of pure counterflow. The corrected reference temperatures are denoted t_i and t_i'.

As a measure of the deviation of the actual flow arrangement from the pure counterflow we take the mean temperature difference. This difference is needed for the design calculations in any case and must be obtained from known equations or diagrams according to [7] through [16].

The corrections ψ and ψ' should decrease the local temperature difference $\Delta T = T - T'$ which would apply in the case of an imaginary counterflow heat exchanger by such a constant factor that the mean temperature difference of the corrected temperatures is that of the actual flow arrangement. Then at each point of the counterflow heat exchanger and thus also at the reference points i = I and II:

$$\frac{\Delta T_i - (\psi_i + \psi_i')}{\Delta T_i} = \frac{\Delta T_M}{\Delta T_{M,c}} \tag{37}$$

where the index c is for counterflow and $\Delta T_{M,c}$ is the logarithmic mean value of $(T_{in} - T_{out}')$ and $(T_{out} - T_{in}')$.

Equation (37) gives a condition only for the *sum* $\psi_i + \psi_i'$ and the problem now is how to distribute the two corrections over the two fluids. The distribution should be done so that the mean corrected counterflow temperatures of both fluids are approximately equal to the real mean temperatures of both fluids. The simplest method of distribution would be to take $\psi_i = \psi_i'$. For symmetric flow arrangements as, for instance, pure crossflow or co-current flow, this equal distribution would be right for $\dot{C} = \dot{C}'$, but for $\dot{C} \neq \dot{C}'$ probably also $\psi_i \neq \psi_i'$. Therefore a function of $\dot{C}'/\dot{C}$ appears reasonable.

$$\frac{\psi_i}{\psi_i'} = \left(\frac{\dot{C}'}{\dot{C}}\right)^{\gamma}. \tag{38}$$

Considering, for instance, a flow arrangement with longitudinal mixing we find, intuitively, that γ is somewhere between zero and one (i.e. the correction of the stream with the stronger temperature change is stronger as well).

To find a reasonable value for γ we consider a most disadvantageous case where the corrections are large. Equation (37) shows that the corrections ψ_i and ψ_i' (compared to ΔT_i) become relatively highest for $\Delta T_M = 0$ and $\Delta T_{M,c} \neq 0$, because then $\psi_i + \psi_i' = \Delta T_i$ and the corrected temperatures become equal and lie somewhere between T and T'.

Co-current flow is the arrangement which differs furthest from countercurrent flow and presents the extreme case for which the correction to be applied to countercurrent flow should be valid.

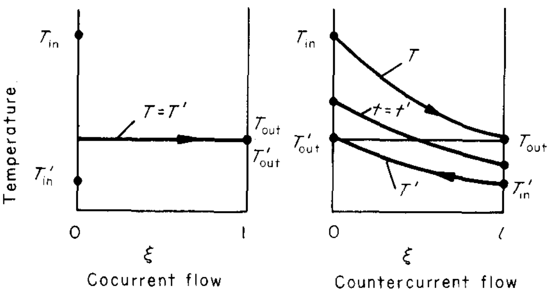

FIG. 4. Temperatures for cocurrent flow with $\Delta T_M = 0$ and for the equivalent countercurrent flow arrangement with temperature correction.

Figure 4 shows the fluid temperatures of an actual cocurrent heat exchanger with $\Delta T_M = 0$ ($T_{out} = T_{out}'$) and the equivalent countercurrent heat exchanger with the corrected temperatures $t = t'$. As the ratio ψ/ψ' shall be constant along the flow path ξ it can be determined by the mean temperatures, integrated over ξ

$$\frac{\psi}{\psi'} = \frac{T_M - T_{out}}{T_{out}' - T_M'} \tag{39}$$

where $T_{out} = T_{out}' = t_M = t_M'$ because $\Delta T_M = 0$.

Because both temperature curves (counterflow) are similar also for changing K

$$\frac{T_M - T_{out}}{T_M' - T_{in}'} = \frac{T_{in} - T_{out}}{T_{out}' - T_{in}'} = \frac{\dot{C}'}{\dot{C}} \tag{40}$$

where $\dot{C}$ and $\dot{C}'$ are constant or the true mean values between the inlet and outlet temperatures [defined by (40)].

The difference of the mean fluid temperature is given by the known equation

$$T_M - T_M' = \frac{(T_{in} - T_{out}') - (T_{out} - T_{in}')}{\ln \dfrac{T_{in} - T_{out}'}{T_{out} - T_{in}}}. \tag{41}$$

From (40) and (41) (with $T_{out} = T_{out}'$) the mean fluid temperatures T_M and T_M' can be expressed as functions of $\dot{C}'/\dot{C}$. Introducing these functions into (39) leads finally to

$$\frac{\psi}{\psi'} = \frac{1-\frac{\dot{C}'}{\dot{C}}+\ln\frac{\dot{C}'}{\dot{C}}}{1-\frac{\dot{C}}{\dot{C}'}+\ln\frac{\dot{C}}{\dot{C}'}}. \tag{42}$$

As shown in Fig. 5 (42) can be approximated very well by an exponential function according to (38) with the exponent $\gamma = \frac{2}{3}$. (This exponent can be derived by series developments of the logarithmic functions in (42) at the point $\dot{C} = \dot{C}'$.)

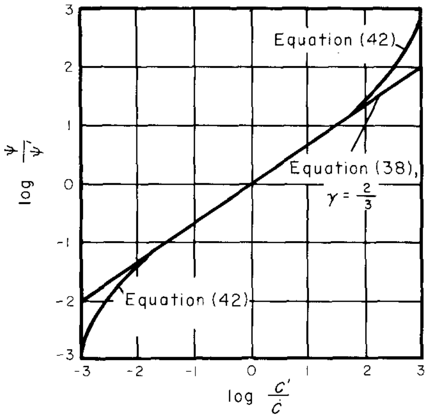

FIG. 5. Comparison between equation (42) and equation (38) with $\gamma = \frac{2}{3}$.

The inaccuracy of the exponential function for large and small values of $\dot{C}'/\dot{C}$ is unimportant for our approximation because then also the corrections ψ and ψ' become small ($\psi+\psi' \to 0$ for $\dot{C}'/\dot{C} \to \infty$ or 0).

Now, combining (37) and (38) with $\gamma = \frac{2}{3}$ we find (for i = I and II)

$$\psi_i = \Delta T_i \cdot \frac{1-\Delta T_M/\Delta T_{M,c}}{1+(\dot{C}/\dot{C}')^{2/3}}$$
$$\psi'_i = \Delta T_i \cdot \frac{1-\Delta T_M/\Delta T_{M,c}}{1+(\dot{C}'/\dot{C})^{2/3}}. \tag{43}$$

With (43) the reference temperatures t_{I}, t_{II}, t'_{I} and t'_{II} according to (36) can be determined.

If laminar flow does not occur, the local overall transfer coefficients K_{I} and K_{II} can now be calculated according to (2) or (4); no length effect is present.

3.2 *Calculation of local laminar heat-transfer coefficients*

With laminar flow, however, only (2) can be applied together with (3) for the local laminar heat-transfer coefficient. The correction δ for laminar length effect can be calculated according to [6]. For the hot stream $\dot{C}$:

$$\delta_{\mathrm{I}} = \tfrac{2}{3}\cdot\left(\frac{\tilde{K}_{\mathrm{II}}}{\tilde{K}_{\mathrm{I}}}\right)^{0\cdot342} + \frac{0\cdot43}{\frac{1}{a_{\mathrm{I}}}+2\cdot3}$$
$$\delta_{\mathrm{II}} = \tfrac{4}{3}\cdot\left(\frac{\tilde{K}_{\mathrm{II}}}{\tilde{K}_{\mathrm{I}}}\right)^{0\cdot092} + \frac{0\cdot1}{\frac{1}{a_{\mathrm{II}}}+1\cdot9} \tag{44}$$

and for the cold stream $\dot{C}'$:

$$\delta'_{\mathrm{I}} = \tfrac{4}{3}\cdot\left(\frac{\tilde{K}_{\mathrm{I}}}{\tilde{K}_{\mathrm{II}}}\right)^{0\cdot092} + \frac{0\cdot1}{\frac{1}{a'_{\mathrm{I}}}+1\cdot9}$$
$$\delta'_{\mathrm{II}} = \tfrac{2}{3}\cdot\left(\frac{\tilde{K}_{\mathrm{I}}}{\tilde{K}_{\mathrm{II}}}\right)^{0\cdot342} + \frac{0\cdot43}{\frac{1}{a'_{\mathrm{II}}}+2\cdot3}. \tag{45}$$

The original constants 0·690 and 1·332 of (77) in [6] have been changed to $\frac{2}{3}$ and $\frac{4}{3}$ improving the accuracy for mean heat-transfer calculations. (Pressure drop calculation is effected weakly; the example Table 3 in [6] yields also better results with the new constants.)

Before the local overall heat-transfer coefficient according to (2) can be calculated the wrong traditional mean value according to (4) must be determined, which appears in (44) and (45).

3.3 *The mean overall heat-transfer coefficient and total pressure drop*

Once the two local overall heat-transfer coefficients have been calculated we find according to the counter-flow method [4] the mean value needed for the design:

$$\frac{1}{K_M} = \frac{1}{2}\left[\frac{1}{K_{\mathrm{I}}}+\frac{1}{K_{\mathrm{II}}}\right]. \tag{46}$$

For the calculation of the pressure drop for any flow arrangements the same considerations are valid as for heat transfer and thus with the corrected reference temperatures also the total pressure drop can be calculated according to [6].

For the hot stream $\dot{C}$

$$\Delta p = \frac{\Delta p_{\mathrm{I}}/K_{\mathrm{I}}+\Delta p_{\mathrm{II}}/K_{\mathrm{II}}}{1/K_{\mathrm{I}}+1/K_{\mathrm{II}}} \tag{47}$$

where Δp_{I} and Δp_{II} are the local total pressure drops calculated with the local properties. For liquids Δp according to (47) is the final pressure drop (inlet and exit losses are not included and must be added).

For gases, however, this pressure drop must be corrected for pressure dependence of density. Now Δp according to (47) is denoted with $\Delta p_{(\mathrm{in})}$ or $\Delta p_{(\mathrm{out})}$ because it had to be determined with the pressure at inlet or outlet, respectively (however with t_{I} and t_{II}).

For gases the final pressure drop is now:

$$\Delta p = p_{\text{in}}\left[1-\left(1-\frac{2\Delta p_{(\text{in})}}{p_{\text{in}}}\right)^{1/2}\right]$$
$$= p_{\text{out}}\left[\left(1+\frac{2\Delta p_{(\text{out})}}{p_{\text{out}}}\right)^{1/2}-1\right]. \quad (48)$$

When p is replaced by p' (47) and (48) can be applied also for the cold stream.

Thus the pressure drop and the true mean overall heat-transfer coefficient (needed for calculation of the area) can be calculated allowing for changing heat-transfer coefficients due to length and temperature effects for any flow arrangement for which the mean temperature difference can be calculated. The accuracy of this general method shall be tested with some examples in the following part.

4. TEST OF THE GENERAL METHOD

We consider a few limiting cases where either temperature or length effects are present.

4.1 *Only temperature effects*

In the limiting case of pure counterflow the presented general method turns into the known countercurrent method [4] which has been tested sufficiently with excellent results [4, 5]. We now have to consider cases which deviate greatly from the limiting counterflow case.

4.1.1 *Pure cocurrent flow*. This example is unrealistic with respect to practical application because this case would have to be treated according to the special cocurrent method [4] and [6]. However, for testing purposes this example is very useful because of its great deviation from counterflow and because it can be calculated with a high accuracy according to [4].

We imagine the case that a viscous turbulent liquid is cooled by a gas. The heat-transfer coefficient of the liquid will decrease together with its temperature and the heat-transfer coefficient on the gas side will decrease with rising gas temperature. A strong but reasonable dependence of K on T and T' could be

$$K = \frac{5\,.\,T^2}{215+T'} \quad (49)$$

which function was assumed for the three cocurrent examples. The inlet and outlet temperatures of both streams were chosen so that the ratio $\dot{C}/\dot{C}'$ becomes 1/2, 1 and 2 and so that K according to (49) calculated in the conventional way with the arithmetic mean values of inlet and outlet temperatures becomes equal to 100. For the three examples shown in Table 2 the mean overall heat-transfer coefficient K_M was calculated according to four different methods. First, according to the conventional method with arithmetic mean values resulting in $K_M = 100$. Secondly, the flow arrangement was regarded as counterflow. Thirdly, according to the presented general method where the flow arrangement was regarded again as counterflow, but the reference temperatures are corrected. Fourth, according to the two-point cocurrent method described in [4]. For the calculation of the relative error of each method the result of the last special method was regarded as exact (the error of this method is very low, as shown by the examples in [4] and [5]).

The results in Table 2 demonstrate clearly that even in very extreme cases with respect to flow arrangement and temperature dependence (49) the general method yields excellent results compared to the first two methods. Further, one may conclude that omission of *the stipulated correction of the reference temperatures* can lead to a large error.

4.1.2 *One row crossflow*. We now consider similar examples, shown in Table 3, for the more realistic case of crossflow with one row, for which case a special method has been derived in this paper. For the calculation of the relative errors of each method again

Table 2. Test of the general method for heat-transfer coefficients dependent only on temperature. Three examples of cocurrent flow, calculated according to four methods

	Inlet	Outlet	Inlet	Outlet	Inlet	Outlet
T	90	50	85	55	80	60
T'	20	40	15	45	10	50
$\dot{C}/\dot{C}'$	$\frac{1}{2}$		1		2	
	K_M	rel. error %	K_M	rel. error %	K_M	rel. error %
Usual method with arithmetic mean values	100·00	+32·9	100·00	+22·9	100·00	+15·0
Regarded as counter-flow (without correction)	88·44	+17·5	96·33	+18·4	100·50	+15·6
General method, corrected counter-flow	74·65	−0·80	82·07	+0·84	89·26	+2·64
Two-point cocurrent method according to [4]	75·25	0	81·38	0	86·96	0

Table 3. Test of the general method for heat-transfer coefficients dependent only on temperature. Three examples of one row cross-flow, calculated according to four methods

	Inlet	Outlet	Inlet	Outlet	Inlet	Outlet	Inlet	Outlet
T	100	40	100	40	110	30	85	55
T'	0	60	0	60	10	50	0	60
$\dot{C}/\dot{C}'$	1		1		$\frac{1}{2}$		2	
K	Equation (49)		Equation (50)		Equation (50)		Equation (50)	
	K_M	rel. error %	K_M	rel. error %	K_M	rel. error %	K_M	rel. error %
Usual method with arithmetic mean values	100·00	+58·1	100·00	+58·3	100·00	+132·9	100·00	+10·4
Regarded as counter-flow (without correction)	85·88	+35·8	83·04	+31·4	54·67	+27·3	101·31	+11·8
General method, corrected counter-flow	62·36	−1·4	62·09	−1·73	42·08	−2·01	90·87	+0·34
One row cross-flow method according to 2·2	63·24	0	63·18	0	42·94	0	90·57	0

the last special method was considered to be exact. In the first example with $\dot{C} = \dot{C}'$ K was taken according to (49), and additional iterations were necessary to determine the reference temperatures of T'. In the examples two, three and four, where the effect of changing $\dot{C}/\dot{C}'$ was tested, the dependence of K on T' was neglected for simplification (or taken into account approximately only) by calculating K according to (49), however, with a constant mean value $T' = 30$.

$$K = \frac{T^2}{49}. \tag{50}$$

The results obtained using (50) instead of (49) are very similar (see examples one and two) and yield virtually the same results.

Again, the general method yields good results and the reference temperature correction can have a decisive effect. The results for different ratios $\dot{C}/\dot{C}'$ also demonstrate that the distribution of the corrections ψ and ψ' according to (38) with $\gamma = \frac{2}{3}$ has the desired effect.

4.2 *Laminar length effects only*

We consider the cases which have been treated and discussed in detail in Section 2.1.

4.2.1 *One stream laminar.* In Section 2.1.1 (9) was derived for the ratio V. With the same assumptions we now derive an equation for V according to (6) using the general method described in Section 3. In (6) $\bar{K}$ must be substituted according to (4) in which the variable a according to (7) must be introduced. The mean value K_M has to be calculated using (2), (3), (7), (44) and (46). Then we find finally:

$$\frac{1}{V} = 1 + \frac{1}{1+a}\left(\frac{0{\cdot}215}{1/a+2{\cdot}3} + \frac{0{\cdot}05}{1/a+1{\cdot}9}\right). \tag{51}$$

The limiting values of V according to this equation compare with those of (9), for $a \to 0$ or ∞ $V = 1$. Table 4 shows other values calculated from (9) and (51), respectively. The agreement of both equations is excellent. The relative error between both values of V is equal to that of the true mean overall heat-transfer coefficient K_M according to the general approximation method.

Thus the derived general method is valid for any flow arrangement if one stream is laminar.

Table 4. Test of the general method for length effect and one laminar stream. Comparison between equation (9) and equation (51)

a	V, equation (9)	V, equation (51)	Error (rel.) %
0·1	0·98044	0·98067	+0·023
0·2	0·97131	0·97033	−0·101
0·5	0·96210	0·95980	−0·238
0·7	0·96124	0·95900	−0·233
1·0	0·96229	0·96043	−0·193
2·0	0·96924	0·96849	−0·077
5·0	0·98190	0·98203	+0·013
10·0	0·98945	0·98969	+0·024

4.2.2 *Both streams laminar.* Corresponding to the derivation of (12) in Section 2.1.2 we now derive with the general method and in the same way as for one laminar stream in the previous section but using, in addition, (45):

$$\frac{1}{V} = 1 + \frac{0{\cdot}215}{1+1/a+2{\cdot}3(1+a)} + \frac{0{\cdot}215}{1+a+2{\cdot}3(1+1/a)} + \frac{0{\cdot}05}{1+1/a+1{\cdot}9(1+a)} + \frac{0{\cdot}05}{1+a+1{\cdot}9(1+1/a)}. \tag{52}$$

This V again has the same limiting value 1 for $a \to \infty$

or 0 as V according to (12). Table 5 gives a comparison of (12) and (52). The agreement is again excellent. Thus with two laminar streams and any kind of crossflow (as could occur in compact heat exchangers) the general method will yield good results.

Table 5. Test of the general method for length effect and two laminar streams. Comparison between equation (12) and equation (52)

	a	V, equation (12)	V, equation (52)	Error (rel.) %
0·1	10·0	0·96909	0·97075	+0·172
0·2	5·0	0·95309	0·95339	+0·032
0·5	2·0	0·93307	0·93073	−0·251
1·0	1·0	0·92711	0·92388	−0·348

With pure counterflow and two laminar streams, for which case the general method also would yield (52), the correction factor V and K_M [see (6)] is slightly too high as shown in Fig. 2 for zero wall resistance. The general method may yield a K_M which is up to 3·8 per cent too high in the most disadvantageous case of $\alpha_L = \alpha'_L$ and $w = 0$. (Pressure drop is not affected as only the ratio of the two local overall heat-transfer coefficients is decisive, as discussed in [6].) *However, pure countercurrent flow does not occur frequently in practical design.*

With pure cocurrent flow our general method would yield values of K_M which are slightly too low, which is obvious for zero wall resistance where $V = 1$.

Thus for the practical flow arrangements where cocurrent flow, countercurrent flow and crossflow are combined (e.g. shell and tube heat exchangers) the errors of the cocurrent and countercurrent streams cancel and the general method will yield good results also even for the (seldom encountered) case of two laminar streams.

5. CONCLUSION

For all practical purposes the general method developed for the thermal design of a single phase heat exchanger gives reliable results and provides a major advantage over conventional methods.

For some special cases of crossflow more specific methods can be used for the calculation of the mean overall heat-transfer coefficient.

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APPENDIX

For the integration of (11) the following integral must be used which can be solved by partial integration and which is given, for instance, in [17] p. 94, number 4.

$$\int x^n . \ln x . dx = \frac{x^{n+1}}{n+1} . \left(\ln x - \frac{1}{n+1}\right) + B. \qquad \text{(A1)}$$

The integration of (11) can be done as follows. Integrating only the first two terms of the sum under the integral gives

$$V = (1+a) . (1 - \tfrac{3}{2} . a + 2 . a^2 . I) \qquad \text{(A2)}$$

where

$$I = \int_{\phi=0}^{\phi=1} \phi^{2/3} . \ln\left(1 + \frac{1}{a . \phi^{1/3}}\right) . d\phi. \qquad \text{(A3)}$$

By extracting $1/a . \phi^{1/3}$ out of the brackets the logarithm, and thus the integral, can be broken down in three integrals

$$I = I_x + I_y + I_z \qquad \text{(A4)}$$

where the first gives

$$I_x = -\ln a . \int_{\phi=0}^{\phi=1} \phi^{2/3} . \mathrm{d}\phi = -\tfrac{3}{5} . \ln a \qquad \text{(A5)}$$

and the second one can be solved according to (A1) with $x = \phi$ and $n = \frac{2}{3}$

$$I_y = -\tfrac{1}{3} \int_{\phi=0}^{\phi=1} \phi^{2/3} . \ln \phi . \mathrm{d}\phi = \tfrac{3}{25}. \qquad \text{(A6)}$$

The third integral to be solved is

$$I_z = \int_{\phi=0}^{\phi=1} \phi^{2/3} . \ln(1 + a . \phi^{1/3}) . \mathrm{d}\phi. \qquad \text{(A7)}$$

Substituting ϕ according to

$$x = 1 + a . \phi^{1/3} \qquad \text{(A8)}$$

gives:

$$I_z = \frac{3}{a^5} . \int_{x=1}^{x=1+a} (x-1)^4 . \ln x . \mathrm{d}x \qquad \text{(A9)}$$

and with the binomial series

$$I_z = \frac{3}{a^5} . \int_{x=1}^{x=1+a} (x^4 - 4.x^3 + 6.x^2 - 4.x + 1) . \ln x . \mathrm{d}x. \qquad \text{(A10)}$$

This integral can be broken down into five integrals which can be solved according to the integral (A1) with $n = 4, 3, 2, 1$ and 0. This yields finally:

$$I_z = \frac{3}{5} . \ln(1+a) - \frac{3}{25} + \frac{3}{20.a} - \frac{1}{5.a^2} + \frac{3}{10.a^3} - \frac{3}{5.a^4} + \frac{3}{5.a^5} . \ln(1+a). \qquad \text{(A11)}$$

Substituting in (A4) I_z, I_y and I_x according to (A11), (A6) and (A5) and introducing I into (A2) leads to (12) of this paper.

CALCUL D'UN ECHANGEUR DE CHALEUR AVEC COEFFICIENTS DE TRANSFERT VARIABLES POUR DES ARRANGEMENTS A ECOULEMENTS CROISES ET MIXTES

Résumé—Dans des cas particuliers d'écoulements croisés, on peut calculer le coefficient global de transfert thermique en prenant en compte soit les effets de longueur laminaire, soit la dépendance à la température. Pour un écoulement laminaire la valeur moyenne peut être calculée, en écoulements à cocourant ou à contre-courant, et le résultat est valable pour un arrangement quelconque. Dans le cas de deux écoulements laminaires, on présente une solution valable pour un arrangement croisé quelconque.

On présente une méthode spéciale relative à la dépendance du transfert thermique vis à vis de la température et aux écoulements croisés, pour un seul rang. Selon cette méthode, le transfert thermique doit être calculé en deux points s'il dépend seulement de la température du fluide dans le tube, et en quatre points s'il est fonction des températures des deux fluides.

Une méthode approchée plus générale est décrite pour calculer le coefficient global de transfert et la chute de pression globale, méthode valable pour un arrangement quelconque et des effets combinés de longueur et de température. Cette approche est développée à partir d'une étude antérieure de contre-courant pur; le transfert thermique et la chute de pression doivent être calculés en deux points.

La méthode générale est testée sur deux exemples dont on connait les solutions (cas de courants croisés et de cocourant) et un très bon accord est constaté.

DIE AUSLEGUNG VON KREUZSTROM- UND MISCHSTROM-WÄRMEÜBERTRAGERN BEI VERÄNDERLICHEN WÄRMEDURCHGANGSKOEFFIZIENTEN

Zusammenfassung—Für spezielle Fälle des Kreuzstroms kann der mittlere Wärmedurchgangskoeffizient berechnet werden, wenn entweder die Abhängigkeit des Durchgangskoeffizienten vom Strömungsweg oder von der Temperatur bekannt ist. Wenn ein Strom laminar ist, kann der mittlere Wärmedurchgangskoeffizient in gleicher Weise wie für Gleich- und Gegenstrom berechnet werden, wobei die Ergebnisse für jede Art der Strömungsführung gültig sind. Für den Fall, daß beide Ströme laminar sind, wird ein Verfahren angegeben, welches auf jede Art von Kreuzstrom angewendet werden kann.

Für nur temperaturabhängigen Wärmeübergang und für Kreuzstrom mit nur einer Rohrreihe wird ein spezielles Verfahren angegeben. Hängt der Wärmeübergang nur von der Fluidtemperatur im Rohr ab, so ist die Rechnung für zwei Punkte durchzuführen. Sind beide Fluidtemperaturen von Einfluß, so ist eine Rechnung für vier Punkte erforderlich. Weiterhin wird für die Berechnung des mittleren Wärmedurchgangskoeffizienten und den Druckabfall ein allgemein gültiges Näherungsverfahren vorgeschlagen, welches für jede Strömungsführung und auch bei Weg- und Temperaturabhängigkeit der Koeffizienten anwendbar ist. Das Näherungsverfahren wurde aus einer für reinen Gegenstrom entwickelten Methode abgeleitet. Der Wärmedurchgang und der Druckabfall muß für zwei Stellen berechnet werden. Das Verfahren wurde an verschiedenen, bereits gelösten Anwendungsfällen (speziellen Formen von Kreuzstrom und Gleichstrom) erprobt, wobei sehr gute Übereinstimmung festzustellen war.

КОНСТРУКЦИЯ ТЕПЛООБМЕННИКОВ С ПЕРЕМЕННЫМИ КОЭФФИЦИЕНТАМИ ПЕРЕНОСА ДЛЯ СИСТЕМ С ПОПЕРЕЧНЫМ И СМЕШАННЫМ ТЕЧЕНИЯМИ

Аннотация — В частном случае перекрестного тока может быть рассчитан суммарный средний коэффициент теплообмена, когда имеет место влияние либо длины ламинарного потока, либо температурной зависимости. При наличии только одного ламинарного потока можно рассчитать среднее значение коэффициента теплопередачи для прямоточного и противоточного течений; полученный результат справедлив для любых течений. В случае двух ламинарных течений приводится решение для любых структур перекрестного тока.

Если теплообмен зависит только от температуры, то для случая перекрестного тока с одним рядом труб приводится специальный метод расчета. Согласно данному методу необходимо рассчитывать теплообмен в двух точках в случае его зависимости только от температуры жидкости внутри трубы и в четырех точках, если он является функцией обеих температур жидкости.

Далее описывается более общий приближенный метод расчета среднего суммарного коэффициента теплообмена и суммарного перепада давления, применяемого для любых типов течений. Это приближение развито на основе более раннего метода расчета чисто противоточного течения, при этом теплообмен и перепад давлений должны рассчитываться в двух точках.

Общий метод проверялся на примерах, для которых существуют надежные решения (особые случаи перекрестного и прямоточного течений). Получено хорошее совпадение между теорией и экспериментом.